

# $\mathcal{N} = 2$ Supersymmetry in a Hybrid Inflation Model

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## Abstract

The slow roll inflation requires an extremely flat inflaton potential. The supersymmetry (SUSY) is not only motivated from the gauge hierarchy problem, but also from stabilizing that flatness of the inflaton potential against radiative corrections. However, it has been known that the Planck suppressed higher order terms in the Kähler potential receive large radiative corrections loosing the required flatness in the  $\mathcal{N} = 1$  supergravity. We propose to impose a global  $\mathcal{N} = 2$  SUSY on the inflaton sector. What we find is that the  $\mathcal{N} = 2$  SUSY Abelian gauge theory is exactly the same as the desired hybrid inflation model. The flat potential at the tree level is not our choice of parameters but a result of the symmetry. We further introduce a cut-off scale of the theory which is lower than the Planck scale. This lower cut-off scale suppresses the supergravity loop corrections to the flat inflaton potential.

Inflationary universe is widely believed as the standard theory of modern cosmology [1], since there has been found, so far, no alternative to solve the fundamental problems in cosmology, i.e. the horizon and the flatness problems. The inflationary cosmology leads to a very flat universe ( $\Omega \simeq 1$ ) at the present time, and this remarkable prediction has been strongly supported by recent observations on cosmic microwave background radiations (CMBR) by Boomerang [2] and Maxima [3]. The inflationary universe requires a scalar field called as inflaton  $\varphi$  which should have necessarily a very flat potential to generate the inflation in the early universe. However, this flat potential is subject to radiative corrections and it receives easily a large deformation losing the required flatness. Supersymmetry (SUSY) is an interesting theory which may protect the flat potential for the inflaton  $\varphi$  from having large radiative corrections. In fact, the superpotential is completely stable against the radiative corrections due to the nonrenormalization theorem [4]. However, the Kähler potential receives large radiative corrections even in the SUSY theories [5] and we need a fine-tuning of various parameters in supergravity to maintain the flatness of inflaton potential at the quantum level [6].

A possible solution to the above problem is to introduce a cut-off scale  $M_*$  much below the Planck scale  $M_{\text{Planck}} \simeq 2 \times 10^{18}$  GeV in order to suppress the unwanted radiative corrections in the Kähler potential for inflaton  $\varphi$ . We adopt the cut-off scale  $M_* \ll M_{\text{Planck}}$  throughout this paper. However, the introduction of the cut-off raises a new serious problem; nonrenormalizable operators of the inflaton field  $\varphi$  are suppressed by inverse powers of the cut-off scale  $M_*$  which easily destroys the flatness of the scalar potential  $V$  because of  $M_* \ll M_{\text{Planck}}$ . The situation becomes even worse.

In this paper, we propose to use a  $\mathcal{N} = 2$  SUSY to control the tree-level Kähler and superpotentials for the inflaton and construct an explicit model (the hybrid inflation model) which may avoid the above mentioned fine-tuning problem. In this model the form of Kähler and superpotentials are fixed by the global  $\mathcal{N} = 2$  SUSY and the additional R-symmetry. In particular, we show that the tree-level Kähler potential has necessarily the minimal form and hence it is independent of the cut-off scale  $M_*$ . Thus, the flat potential is guaranteed by the symmetries. The gravity interactions, however, break explicitly the global  $\mathcal{N} = 2$  SUSY, since we impose only the  $\mathcal{N} = 1$  supergravity in the full theory. We calculate radiative corrections from the supergravity sector and find that they are sufficiently suppressed owing to the small cut-off scale  $M_*$ . Thus, we consider that our hybrid inflation model based on the global  $\mathcal{N} = 2$  SUSY is perfectly natural.

Before describing our explicit model we first discuss our basic assumptions. First of all, we adopt the framework of  $\mathcal{N} = 1$  supergravity for the full theory whose scalar potential is determined by two fundamental potentials, i.e. Kähler potential  $K(\Phi^{\dagger\bar{i}}, \Phi^i)$  and superpotential  $W(\Phi^i)$ , where  $\Phi^i$  are chiral superfields. The scalar potential is given by

$$V = \exp\left(\frac{K(\phi^\dagger, \phi)}{M_{\text{Planck}}^2}\right) \left[ \left(W_i + \frac{K_i}{M_{\text{Planck}}^2} W\right) \left(W_j^\dagger + \frac{K_{\bar{j}}}{M_{\text{Planck}}^2} W^\dagger\right) K^{\bar{j}i} - 3 \left|\frac{W(\phi)}{M_{\text{Planck}}}\right|^2 \right], \quad (1)$$

where

$$W_i = \frac{\partial W(\phi)}{\partial \phi^i}, \quad W_j^\dagger = \frac{\partial W^\dagger(\phi^\dagger)}{\partial \phi^{\dagger \bar{j}}}, \quad (2)$$

$$K_i = \frac{\partial K(\phi, \phi^\dagger)}{\partial \phi^i}, \quad K_{\bar{j}} = \frac{\partial K(\phi, \phi^\dagger)}{\partial \phi^{\dagger \bar{j}}}, \quad (3)$$

$$K_{i\bar{j}} = \frac{\partial^2 K(\phi, \phi^\dagger)}{\partial \phi^i \partial \phi^{\dagger \bar{j}}}, \quad K_{i\bar{j}}(\phi, \phi^\dagger) K^{\bar{j}k}(\phi, \phi^\dagger) = \delta_i^k. \quad (4)$$

Here,  $\phi^i, \phi^{\dagger \bar{j}}$  are scalar components of the (anti-) chiral superfields  $\Phi^i, \Phi^{\dagger \bar{j}}$ . The first nontrivial assumption is that the Kähler potential  $K$  and superpotential  $W$  do not depend on the Planck scale  $M_{\text{Planck}}$  but depend on the cut-off scale  $M_*$ .<sup>1</sup> This means that the basic interactions for matter superfields  $\Phi^i$  are controlled only by the cut-off scale  $M_*$  instead of the Planck scale  $M_{\text{Planck}}$ . In particular, the metric of the scalar components  $\phi^i$  is given by the Kähler potential as

$$\mathcal{L} = K_{i\bar{j}}(\phi, \phi^\dagger) \partial_\mu \phi^{\dagger \bar{j}} \partial^\mu \phi^i, \quad (5)$$

and hence the metric of the scalar fields  $\phi^i$  is controlled by the cut-off scale  $M_*$ . We stress that this is a necessary assumption to obtain the desired flat potential as we will see below. It is beyond the scope of this paper to discuss the underlying physics that may justify this assumption.

The second assumption is rather simple, which is that the Lagrangian for the inflaton sector possesses a global  $\mathcal{N} = 2$  SUSY in the limit of  $M_{\text{Planck}} = \infty$ . This assumption implies that when one switches off the  $\mathcal{N} = 1$  supergravity interactions, the inflaton sector is completely decoupled from other sectors including the standard-model fields. This assumption together with the previous one means that the field theory described by the Kähler and superpotentials,  $K$  and  $W$ , for the inflaton sector alone is invariant under the global  $\mathcal{N} = 2$  SUSY, since they are independent of the Planck scale  $M_{\text{Planck}}$ .

We now discuss the inflaton sector which possesses the global  $\mathcal{N} = 2$  SUSY. We consider a  $U(1)$  gauge theory with a hypermultiplet ( $\Psi$  and  $\bar{\Psi}$ ).  $\Psi$  and  $\bar{\Psi}$  are charged under the  $U(1)$  gauge symmetry with the charge  $+1$  and  $-1$ , respectively.

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<sup>1</sup> Here, we suppose that the Kähler potential  $K$  has a fundamental meaning rather than the  $-3M_{\text{Planck}}^2 \exp(-K/(3M_{\text{Planck}}^2))$ . The former describes the metric of manifold of matter superfields. Its geometrical meaning is clear. The latter is a natural expression in the superspace Lagrangian of the supergravity [7].

The general superpotential of the  $\mathcal{N} = 2$   $U(1)$  gauge theory are [8]

$$W = \sqrt{2}(\bar{\Psi}\Phi\Psi - \mu^2\Phi), \quad (6)$$

where  $\Phi$  is the  $\mathcal{N} = 1$  chiral superfield which is a partner of the  $\mathcal{N} = 1$   $U(1)$  gauge vector multiplet in the  $\mathcal{N} = 2$  SUSY theory. The first term is one of the “gauge interactions”, and forms a  $\mathcal{N} = 2$  theory along with the ordinary  $\mathcal{N} = 1$  gauge interaction in the Kähler potential. The second term is one of the “Fayet-Iliopoulos term”. This term  $-\sqrt{2}\mu^2\Phi|_{\theta^2}$ , along with the ordinary Fayet-Iliopoulos  $D$ -term  $\mathcal{L}_{\text{FI-D}} = -\xi^2 D$ , forms a  $SU(2)_R$  singlet  $\mathcal{L}_{\text{FI}} = -(2\text{Im}\mu^2, 2\text{Re}\mu^2, \xi^2) \cdot (-\sqrt{2}\text{Im}F, \sqrt{2}\text{Re}F, D)^T$  [8]. The  $SU(2)_R$  symmetry is explicitly broken by the  $SU(2)_R$  triplet vacuum-expectation value  $(2\text{Im}\mu^2, 2\text{Re}\mu^2, \xi^2)$ . However, this superpotential (6) is invariant under the  $U(1)_R$  symmetry, where the  $R$ -charges of  $\Phi, \Psi$  and  $\bar{\Psi}$  are 2, 0 and 0, respectively. We impose the  $U(1)_R$  symmetry throughout this paper.

The Kähler potential for  $\Phi$  is determined from the prepotential  $\mathcal{F}(\Phi, M_*)$  as [9]

$$K(\Phi, \Phi^\dagger, M_*) = \frac{1}{4\pi} \text{Im} \left( \Phi^\dagger \frac{\partial}{\partial \Phi} \mathcal{F}(\Phi, M_*) \right). \quad (7)$$

Here, the prepotential  $\mathcal{F}(\Phi, M_*)$  is a holomorphic function of the superfield  $\Phi$  carrying  $R$ -charge 4 and hence the  $R$ -invariance requires

$$\mathcal{F}(\Phi, M_*) = \frac{1}{2} \Phi^2 \left( \frac{\vartheta}{2\pi} + i \frac{4\pi}{g^2} \right), \quad (8)$$

which leads to the minimal Kähler potential for  $\Phi$ ,

$$K = \frac{1}{g^2} \Phi^\dagger \Phi. \quad (9)$$

Here,  $g$  is the  $U(1)$  gauge coupling constant. We should stress that the minimal Kähler potential for  $\Phi$  is not our choice by hand, but rather a result of the symmetries, i.e. the  $\mathcal{N} = 2$  SUSY and the  $R$ -invariance. This is a big merit of the  $\mathcal{N} = 2$  SUSY. Notice that the Kähler potential of  $\Phi$  is independent not only of  $M_{\text{Planck}}$  but also of the cut-off scale  $M_*$ .

The renormalization of the chiral superfield  $\Phi$  is necessary to obtain the canonically normalized kinetic term,

$$K = \Phi^\dagger \Phi. \quad (10)$$

Then, the superpotential (6) becomes

$$W = \sqrt{2}g\Phi(\Psi\bar{\Psi} - \mu^2). \quad (11)$$

It is very remarkable that the superpotential (11) along with the minimal Kähler potential (10) of  $\Phi$  is exactly the same as that in the hybrid inflation model [10,11], where one of the scalar component  $\phi$  of the  $\Phi$  superfield plays a role of the inflaton  $\varphi$ .

On the other hand, the Kähler potential for hypermultiplet ( $\Psi$  and  $\bar{\Psi}$ ) contains non-renormalizable terms. Indeed the Kähler manifold of hypermultiplet is a hyperKähler manifold, and it is not determined by the special geometry as in eq.(7). The hyperKähler condition allows

$$K(\Psi, \bar{\Psi}, M_*) = \Psi^\dagger \Psi + \bar{\Psi}^\dagger \bar{\Psi} + \frac{k}{4M_*^2} \left( (\Psi^\dagger \Psi)^2 - 4(\Psi^\dagger \Psi \bar{\Psi}^\dagger \bar{\Psi}) + (\bar{\Psi}^\dagger \bar{\Psi})^2 \right) + \dots, \quad (12)$$

where  $k$  is a real constant, and ellipses represent higher order terms. We see that  $\psi = \bar{\psi} = 0$  is at least a local minimum of the scalar potential for  $\psi$  and  $\bar{\psi}$  during the inflation. Here,  $\psi$  and  $\bar{\psi}$  are scalar components of the hypermultiplet ( $\Psi$  and  $\bar{\Psi}$ ). The origin  $\psi = \bar{\psi} = 0$  may not always be the absolute minimum of the scalar potential for  $\psi$  and  $\bar{\psi}$ . However, in the following analysis, we assume that the  $\psi = \bar{\psi} = 0$  is the absolute minimum<sup>2</sup>, and  $\psi$  and  $\bar{\psi}$  stay at the origin  $\psi = \bar{\psi} = 0$  during the inflation.

The tree-level Kähler potential for the inflaton sector may have a deformation due to the quantum effects. If it is too large we lose the flat potential for the inflaton  $\varphi$  and a sufficiently long inflation is not expected.

First, we discuss the radiative corrections through the  $\mathcal{N} = 2$  SUSY interactions. At quantum level, the  $U(1)_R$  symmetry is broken by  $U(1)_R$ -( $U(1)$  gauge)<sup>2</sup> anomaly. Under the  $U(1)_R$  transformation  $\Psi(\theta) \rightarrow \Psi(e^{i\alpha}\theta)$ ,  $\bar{\Psi}(\theta) \rightarrow \bar{\Psi}(e^{i\alpha}\theta)$  and  $\Phi(\theta) \rightarrow e^{-2i\alpha}\Phi(e^{i\alpha}\theta)$ , the Lagrangian is not invariant but

$$\delta\mathcal{L} = \frac{-4\alpha}{32\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu}. \quad (13)$$

This variation of the Lagrangian, however, can be compensated by the variation according to the transformation of the  $\vartheta$  parameter in the Lagrangian as

$$\frac{\vartheta}{32\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu} \rightarrow \frac{\vartheta + 4\alpha}{32\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu}. \quad (14)$$

Then, we have a symmetry “ $U(1)_R$ ” even at the quantum level [12], under which the gauge coupling spurion transforms as

$$\frac{\vartheta}{2\pi} + i\frac{4\pi}{g^2} \rightarrow \frac{\vartheta + 4\alpha}{2\pi} + i\frac{4\pi}{g^2}, \quad (15)$$

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<sup>2</sup> This is not a fine-tuning of parameters in the Kähler potential (12)

along with the ordinary  $U(1)_R$  transformation of the  $\Psi, \bar{\Psi}$  and  $\Phi$  with R-charge 0,0 and 2.

Whole radiative corrections to the Kähler potential of  $\Phi$  are described by the deformation of the prepotential. The deformation allowed by the “ $U(1)_R$ ” symmetry is

$$\mathcal{F}(\Phi, M_*) = \frac{1}{2}\Phi^2 \left[ \left( \frac{\vartheta}{2\pi} + i\frac{4\pi}{g^2(M_*)} - i\frac{2}{2\pi} \ln \left( \frac{\Phi}{M_*} \right) \right) + c_1 e^{\left( -\frac{8\pi^2}{g^2(M_*)} + i\vartheta \right)} \left( \frac{\Phi}{M_*} \right)^2 + \dots \right]. \quad (16)$$

Note that the  $M_*^{-2} \exp \left( -\frac{8\pi^2}{g^2(M_*)} + i\vartheta \right)$  is charged under the “ $U(1)_R$ ” with the charge  $-4$  (see eq.(15)). The first term in the square bracket [ ] corresponds to the 1-loop renormalization of the gauge coupling. This term alone is invariant under the “ $U(1)_R$ ” transformation. This corresponds to the fact that the renormalizations of the gauge couplings of  $\mathcal{N} = 2$  SUSY gauge theories are 1-loop exact (*i.e.* the corrections from the higher order loops are absent). The second and higher terms in the square bracket [ ] represent non-perturbative corrections. Each terms are also invariant under the “ $U(1)_R$ ” symmetry. The deformed prepotential (16) leads to the Kähler potential

$$K(\Phi, \Phi^\dagger, M_*) = \Phi^\dagger \Phi \left[ \left( \frac{1}{g^2(M_*)} - \frac{1}{8\pi^2} \ln \left( \frac{e\Phi^\dagger \Phi}{M_*^2} \right) \right) + \left( c_1' e^{-\frac{8\pi^2}{g^2(M_*)} + i\vartheta} \left( \frac{\Phi}{M_*} \right)^2 + h.c. \right) + \dots \right], \quad (17)$$

or if canonically normalized,

$$K(\Phi, \Phi^\dagger, M_*) = \Phi^\dagger \Phi \left[ 1 - \frac{g^2(M_*)}{8\pi^2} \ln \left( \frac{e|\Phi|^2}{M_*^2} \right) + \left( c_1' g^2 e^{-\frac{8\pi^2}{g^2(M_*)} + i\vartheta} \left( \frac{\Phi}{M_*} \right)^2 + h.c. \right) + \dots \right]. \quad (18)$$

We can see that the logarithmic correction in the Kähler potential is exactly the 1-loop wave-function renormalization of the  $\Phi$  field coming from the Yukawa interaction with the Yukawa coupling  $\sqrt{2}g$ . This 1-loop renormalization effect for the wave function is already considered in [11]. Now, in this  $\mathcal{N} = 2$  SUSY model, higher order (perturbative) wave-function renormalization is absent from the reason stated above. Therefore, the perturbative radiative corrections are already exactly taken into account. The non-perturbative correction terms have exponentially suppressed coefficients  $\exp(-8\pi^2/g^2) \lesssim 10^{-6900}$  for  $\sqrt{2}g \lesssim 10^{-1}$  suggested from the successful inflation as shown later. Thus, we may safely neglect the non-perturbative terms.

We are now at the point to discuss radiative corrections from the supergravity sector, since it breaks explicitly the  $\mathcal{N} = 2$  SUSY. Here, we see that the small cut-off scale  $M_*$  is a crucial ingredient to suppress the quantum effects.

The total 1-loop corrections to the inflaton potential<sup>3</sup> are given by<sup>4</sup> [5]

$$V_{1\text{-loop}} = \frac{M_*^2}{32\pi^2} \text{Str} \left( m^2(\phi) \right) - \frac{\ln(M_*/\mu)}{32\pi^2} \text{Str} \left( m^4(\phi) \right), \quad (19)$$

where

$$\begin{aligned} \text{Str}(m^2(\phi)) &= 2(N-5)\hat{V}(\phi) + 2(N-1)(m_{3/2}(\phi))^2 - 2R_j^i e^K W_{;i} W^{\dagger;j} + \dots \\ \text{Str}(m^4(\phi)) &= 2(N+21)\hat{V}^2 + 4(N+5)\hat{V}m_{3/2}^2 + 2(N+17)m_{3/2}^4 \\ &\quad + 2e^K \left( W_{;i;j} W^{\dagger;i;j} (2\hat{V} + 3m_{3/2}^2) - 2R_n^m W_{;m} W^{\dagger;n} (\hat{V} + m_{3/2}^2) \right) \\ &\quad + 2e^{2K} \left( 2W^{\dagger;i} W_{;i;j} W^{\dagger;j;k} W_{;k} + 2W_{;i;j} W^{\dagger;j;k} R_n^m W_{;m} W^{\dagger;n} + \dots \right) \\ &\quad + \dots \end{aligned} \quad (20)$$

Here we take the  $M_{\text{Planck}}$  to be unity. The  $\hat{V}$  denotes the tree level scalar potential (1), “ $;$ ” the covariant derivative,  $R_{i\bar{j}k\bar{l}}$  the Riemann curvature determined from the Kähler metric,  $N$  the number of the chiral superfields and  $m_{3/2} = e^{\frac{K}{2}} W$ . For details, see [5].

Let us show an estimation for the supertraces starting from their definitions at first. Masses of order of the Hubble parameter ( $H$ ) are given to all scalar fields( $\tilde{\chi}$ ) in addition to the supersymmetric masses  $M_\chi$  originally exist. Mass matrices of scalar fields ( $\tilde{\chi}$ ) are schematically<sup>5</sup> described as

$$(\tilde{\chi}^*, \tilde{\chi}) \begin{pmatrix} |M_\chi|^2 + H^2 + \left| H \frac{\phi}{M_{\text{Planck}}} \right|^2 & M_\chi^* H \frac{\phi}{M_{\text{Planck}}} \\ M_\chi H \frac{\phi^*}{M_{\text{Planck}}} & |M_\chi|^2 + H^2 + \left| H \frac{\phi}{M_{\text{Planck}}} \right|^2 \end{pmatrix} \begin{pmatrix} \tilde{\chi} \\ \tilde{\chi}^\dagger \end{pmatrix}. \quad (22)$$

Then the supertraces are roughly given by

$$\text{Str}(m^2(\phi)) \sim NH^2 + NH^2 \left| \frac{\phi}{M_{\text{Planck}}} \right|^2 + \dots, \quad (23)$$

$$\text{Str}(m^4(\phi)) \sim (N'|M|^2 H^2 + N''H^4) + (N'|M|^2 H^2 + N''H^4) \left| \frac{\phi}{M_{\text{Planck}}} \right|^2 + \dots, \quad (24)$$

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<sup>3</sup> The total 1-loop corrections to the scalar potential cannot be described only by the renormalization of the Kähler potential. So we discuss the corrections to the whole scalar potential not to the Kähler potential.

<sup>4</sup>This expression includes the 1-loop correction discussed in the above analysis.

<sup>5</sup> The real calculation would not be that simple. There are also  $|M_\chi|^2 |\phi/M_{\text{Planck}}|^2$  contributions in the diagonal elements of this mass matrix. However, these contributions also exist in fermion mass matrices and both contributions cancel out each other after we take the supertraces.

where  $N'$  is the number of heavy particles,  $N''$  that of the light particles ( $N = N' + N''$ ) and  $M$  denotes masses of heavy particles.

We find through an explicit calculation with use of the eqs.(20,21) that the rough estimation given in eqs.(23,24) are indeed correct. We show here the explicit evaluation of the second term in eq.(20) as an example:

$$\begin{aligned}
2(N-1)m_{3/2}^2 &= 2(N-1)e^{\frac{|\phi|^2}{M_{\text{Planck}}^2}+\dots} \left| \frac{\sqrt{2}g\mu^2\phi + \dots}{M_{\text{Planck}}^2} \right|^2 \\
&= 2(N-1)\frac{|\sqrt{2}g\mu^2|^2}{M_{\text{Planck}}^2} \left| \frac{\phi}{M_{\text{Planck}}} \right|^2 \\
&\quad + 2(N-1)\langle m_{3/2} \rangle^2 \left( 1 + \left| \frac{\phi}{M_{\text{Planck}}} \right|^2 + \dots \right) + \dots.
\end{aligned} \tag{25}$$

The first term gives the  $|\phi|^2$  term in eq.(23): note that  $\sqrt{2}g\mu^2/M_{\text{Planck}} \sim H$  and that the  $|\phi|^2$  term from the second term is negligible because  $H \gg \langle m_{3/2} \rangle$  during the inflation.

The total 1-loop corrections to the inflaton potential is now given by

$$\begin{aligned}
V_{1\text{-loop}} &\sim \frac{1}{32\pi^2}NM_*^2H^2 + \frac{1}{32\pi^2}N\left(\frac{M_*}{M_{\text{Planck}}}\right)^2H^2|\phi|^2 \\
&\quad - \frac{\ln(M_*/\mu)}{32\pi^2}N'\left(\frac{M}{M_{\text{Planck}}}\right)^2H^2|\phi|^2 + \dots.
\end{aligned} \tag{26}$$

$M$  denotes heavy particle masses which must be lower than the cut-off scale  $M_*$ . Since we know that the number of all particles is large ( $N \gtrsim 100$ ), the 1-loop suppression factor  $1/(32\pi^2)$  is compensated by this large  $N$ . Then the 1-loop corrections would give a mass term of order of the Hubble parameter to the inflaton potential and would violate the slow roll conditions if the cut-off scale were set to be the Planck scale  $M_{\text{Planck}}$ . Furthermore, the perturbation theory would be no longer valid and the tree level potential would lose its meaning.

In order to suppress the loop corrections sufficiently, we impose

$$N\left(\frac{M_*}{M_{\text{Planck}}}\right)^2 \lesssim 1. \tag{27}$$

This means that the cut-off scale  $M_*$  should be lower than the Planck scale as

$$M_* \lesssim \sqrt{\frac{1}{N}}M_{\text{Planck}} \simeq 10^{-1}M_{\text{Planck}}. \tag{28}$$

The above hybrid inflation model with the 1-loop corrections leads to the same inflaton potential as that discussed in [13]. The Yukawa coupling  $\lambda$  in [13] corresponds to the



$\sqrt{2}g$  in this paper, the coefficient of the mass term  $3H^2|\phi|^2$  is  $k$  in [13] while it is  $\sim (1/32\pi^2)N(M_*/M_{\text{Planck}})^2$  here. The  $\mu^2$  there corresponds to the  $\sqrt{2}g\mu^2$  here. The only difference is the assumption on the reheating processes, but this does not lead to a major difference in the allowed parameter region for the desired inflation. The detailed analysis in [13] shows that the desired hybrid inflations occur for a wide parameter region of  $\lambda \lesssim 1.6 \times 10^{-1}$ ,  $k \lesssim 3 \times 10^{-2}$  and  $\mu \sim 10^{13-15}\text{GeV}$ , which suggests  $g \lesssim 1.1 \times 10^{-1}$ ,  $\mu \simeq (1-20) \times 10^{15}\text{GeV}$  and  $M_* \lesssim 0.2 \times M_{\text{Planck}}$  in this model.<sup>6</sup>

The inflaton sector in this scenario has no interaction term with other sectors (including the standard-model sector) not only in the superpotential but also in the Kähler potential. Thus, the inflaton sector fields decay only through the supergravity interactions. Operators relevant to decays of the scalar fields  $\psi$ ,  $\bar{\psi}$  and  $\phi$  are

$$V = W_\psi \frac{K_{\psi^\dagger}}{M_{\text{Planck}}^2} W^\dagger + h.c. + \dots = \sqrt{2}g\phi \langle \bar{\psi} \rangle \frac{\langle \psi \rangle}{M_{\text{Planck}}^2} (y_t h \tilde{q} \tilde{u})^\dagger + h.c. + \dots, \quad (29)$$

$$V = \exp\left(\frac{K(\psi, \bar{\psi})}{M_{\text{Planck}}^2}\right) |W_h|^2 + \dots = \left(\frac{2\text{Re}(\langle K_\psi \rangle \psi + \langle K_{\bar{\psi}} \rangle \bar{\psi})}{M_{\text{Planck}}^2}\right) |y_t \tilde{q} \tilde{u}|^2 + \dots, \quad (30)$$

$$\begin{aligned} \mathcal{L}/\text{dete}_\mu^a &= K_{i\bar{j}} \bar{\chi}^{\bar{j}} i\bar{\sigma}^\mu \left( \partial_\mu \delta_k^i + \Gamma_{kl}^i \partial_\mu \tilde{\chi}^l - \frac{i}{2} \delta_k^i \frac{1}{M_{\text{Planck}}^2} \text{Im}(K_l \partial_\mu \tilde{\chi}^l) + \dots \right) \chi^k \\ &= -\frac{i}{2} \frac{1}{M_{\text{Planck}}^2} \text{Im}(\langle K_\psi \rangle \partial_\mu \psi + \langle K_{\bar{\psi}} \rangle \partial_\mu \bar{\psi}) (\bar{n} i \bar{\sigma}^\mu n + \dots), \end{aligned} \quad (31)$$

where  $h, \tilde{q}, \tilde{u}$  denote higgs and squarks,  $y_t$  the top-quark Yukawa coupling constant,  $\tilde{\chi}$  and  $\chi$  general scalars and fermions,  $n$  a right handed neutrino, and  $\Gamma_{kl}^i$  the Christoffel symbol determined from the Kähler metric. The coherent oscillation of the  $\phi$  field decays through eq.(29). We can see that the coherent oscillation in the  $\psi, \bar{\psi}$  field space is almost parallel to the  $\text{Re}(\langle K_\psi \rangle \psi + \langle K_{\bar{\psi}} \rangle \bar{\psi})$  direction, which decays into radiation of standard-model sector fields through eq.(30). The decay rates are

$$\Gamma_\phi \sim \frac{1}{8\pi} \left( \sqrt{2}g y_t \left( \frac{\mu}{M_{\text{Planck}}} \right)^2 \right)^2 m_\phi, \quad (32)$$

$$\Gamma_{\text{R}(\psi)} \sim \frac{1}{8\pi} \left( y_t^2 \frac{\mu}{M_{\text{Planck}}^2} \right)^2 m_\psi^3, \quad (33)$$

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<sup>6</sup> One might impose that the inflaton-field value during the inflation is smaller than the cut-off scale  $M_*$ . Then, one finds that the parameter space with  $3 \times 10^{-2} \lesssim \sqrt{2}g$  and  $M_* \lesssim \langle \psi \rangle|_{\text{vac}} \sim (0.1-2) \times 10^{16}\text{GeV}$  are excluded. One will see that the Fayet-Iliopoulos parameter  $\mu^2$  is always smaller than the cut-off  $M_*^2$  in the remaining allowed region.

where  $m_\phi = m_\psi = 2g\mu$ . Notice that all states in the inflaton sector form a massive vector multiplet of  $\mathcal{N} = 2$  SUSY, and hence all have the same mass  $m = 2g\mu$ . These two decay rates are almost the same as the decay rate eq.(34) in [13], and hence the reheating temperature  $T_R$  is given by the Fig.8 in [13]. We see the reheating temperatures  $T_R = 10^{4-9}\text{GeV}$  in most of the parameter region, which are low enough to avoid the overproduction of gravitinos of  $m_{3/2} \sim 1\text{TeV}$  [15]. We may invoke the Affleck-Dine type baryogenesis [16] for such low reheating temperatures<sup>7</sup>. The leptogenesis scenario [17] via thermal production of the right handed neutrinos is marginally possible [18].

However, the energy density of the coherent oscillation of the inflaton fields may be converted into other massive inflaton-sector particles through parametric resonance effects [14] well before the decay to light particles. Not a small fraction of the energy density may now be carried by higgsed vector bosons, massive Dirac fermions, and scalar field  $\text{Re}(\langle\psi\rangle\psi^\dagger - \langle\bar\psi\rangle\bar\psi^\dagger)$ , which all do not have decay operators. This leads to a horrible matter dominated universe after the reheating. If it is the case, we have to introduce small explicit breaking operators of the global  $\mathcal{N} = 2$  SUSY in order for those particles to decay. We introduce two small breaking interactions: one is in the superpotential,

$$W = \epsilon_1 \Phi H_u H_d, \quad (34)$$

and the other is in the kinetic function of gauge multiplets,

$$f = \epsilon_2 W^\alpha W'_\alpha, \quad (35)$$

where  $W_\alpha$  is the field strength tensor of the  $U(1)$  gauge multiplet in the inflaton sector and  $W'_\alpha$  that of the  $U(1)_{B-L}$  symmetry which is also supposed to be gauged and higgsed(spontaneously broken). Then, the decay rates of the  $\mathcal{N} = 1$  massive vector multiplet of the inflaton sector are  $\Gamma \sim (1/8\pi)\epsilon_2^2 m$  and those of the  $\mathcal{N} = 1$  massive chiral multiplets  $\Phi$  and  $(\langle\Psi\rangle\bar\Psi + \langle\bar\Psi\rangle\Psi)$  are not smaller than  $\Gamma \sim (1/8\pi)\epsilon_1^2 m$ . Here, we have assumed that the higgsed  $U(1)_{B-L}$  gauge boson mass is lighter than the mass  $m = m_\psi$  of the inflaton-sector gauge boson. If we require the reheating temperature after the decays of these particles to be larger than the 100GeV, we obtain the breaking parameters  $\epsilon_1, \epsilon_2 \gtrsim 10^{-14}(T_R/100\text{GeV})$ . The effect of these small breakings to the inflaton potential is negligible.

One can identify the  $U(1)$  gauge symmetry of this hybrid inflation sector with the  $U(1)_{B-L}$  symmetry itself. In this case, one must impose  $\mathcal{N} = 2$  SUSY on the whole standard model sector, as was studied in [19]. Then we do not have to introduce the above breaking operators (34,35) to make reheating process successful.

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<sup>7</sup> We assume that the Kähler potential for quark and lepton multiplets,  $q, l$  is complicated enough to have a minimum at the cut-off scale,  $q, l \sim M_*$ , during the hybrid inflation.

In this paper, we have proposed a use of  $\mathcal{N} = 2$  SUSY to control the tree-level Kähler potential for the inflaton  $\varphi$  and constructed a hybrid inflation model based on the  $\mathcal{N} = 2$  SUSY  $U(1)$  gauge theory. We have found that the Kähler potential for the inflaton superfield  $\Phi$  is fixed as the minimal form  $K = \Phi^\dagger \Phi$  by the  $\mathcal{N} = 2$  SUSY and the  $U(1)_R$  symmetry. Thus, the flat potential at the tree level is a consequence of the symmetries. In this model, the radiative corrections from the supergravity interactions are sufficiently suppressed by the low cut-off scale  $M_* \lesssim 10^{-1} M_{\text{Planck}}$ , and the model turns out to be almost the same as that investigated intensively in [13]. The inflation potential in our model is determined by three basic parameters, namely the  $U(1)$  gauge coupling constant  $g$ , Fayet-Iliopoulos parameter  $\mu^2$  and the cut-off scale  $M_*$ .

The global  $\mathcal{N} = 2$  SUSY in this model forbids all mixed terms in the Kähler and superpotentials between the inflaton sector and all other sectors. The reheating takes place only through the supergravity interactions with  $1/M_{\text{Planck}}^2$  suppressed amplitudes. If the parametric resonance effect transfers some part of the coherent oscillation energy to various inflaton-sector particles, then we have to introduce small breaking operators of the  $\mathcal{N} = 2$  SUSY in order to make reheating process sufficiently fast.

We have also assumed that the Kähler potential is determined only by the cut-off scale  $M_*$  and not by  $M_{\text{Planck}}$ . This assumption may be based on the picture that the cut-off scale  $M_*$  is truly the fundamental scale of the theory, which is lower than the  $M_{\text{Planck}}$ . Here,  $M_{\text{Planck}}^2$  may be merely an effective scale appearing in the effective 4 dimensional gravity below the fundamental scale. The physics underlying our assumption is under investigation.

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